EXPERIMENTAL INVESTIGATION OF THE ATTENUATION FACTOR OF A WAVE IN A PULSATING TURBULENT FLOW OF GAS IN A CYLINDRICAL CHANNEL CLOSE TO THE FIRST RESONANCE

B. M. Galitseiskii, Yu. A. Ryzhov, and E. V. Yakush

UDC 536.242

The results of measurements of the attenuation factor of the amplitude of pressure and velocity oscillations in a pulsating nonisothermal turbulent flow of air in a cylindrical tube close to the first resonance harmonic (80 ± 5 Hz) are presented.

This work was carried out in order to obtain a better understanding of the effect of pressure and velocity pulsations on hydrodynamic turbulent flow.

As shown in [1], the heat-exchange factor along the length of the channel under resonance oscillation conditions is a variable quantity, and to determine it, it is necessary to know the distribution of the amplitudes of the pressure and velocity oscillations along a wavelength. Recently developed methods of calculating small oscillations have been devoted to isothermal flows, the use of which in heating and cooling can lead to considerable errors.

The nonstationary motion of a gas is described in the one-dimensional formulation by the one-dimensional equations of motion, continuity, energy, and state. We will assume that in each cross section of the channel the propagation of small oscillations is isoentropic. This assumption is quite justified for high-frequency oscillations for which the thickness of the oscillating layer $\delta_k \sim \sqrt{2\nu/\omega}$ is much less than the radius of the channel. Then for an ideal gas the equations of motion, continuity, and state in the one-dimensional formulation can be written in the form

$$\frac{\partial(\rho u)}{\partial\tau} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial P}{\partial x} - \Phi = 0,$$
$$\frac{\partial\rho}{\partial\tau} + \frac{\partial(\rho u)}{\partial x} = 0, \tag{1}$$
$$\frac{dP}{d\rho} = a^2.$$

The parameters of the gas can be represented in the form of the sum of quantities averaged with respect to time and pulsating components.

We will assume that the pulsation force of friction on the wall $\Delta\Phi_w$ can be represented in the form of the linear relationship

$$\Delta\Phi_w = -m\Delta(\rho u),$$

where m is the loss factor which characterizes the amplitude of the frictional oscillations on the walls as a function of the amplitude of the oscillations $\Delta(\rho u)$. With these assumptions the system of initial equations

S. Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 29, No. 1, pp. 31-38, July, 1975. Original article submitted January 17, 1975.

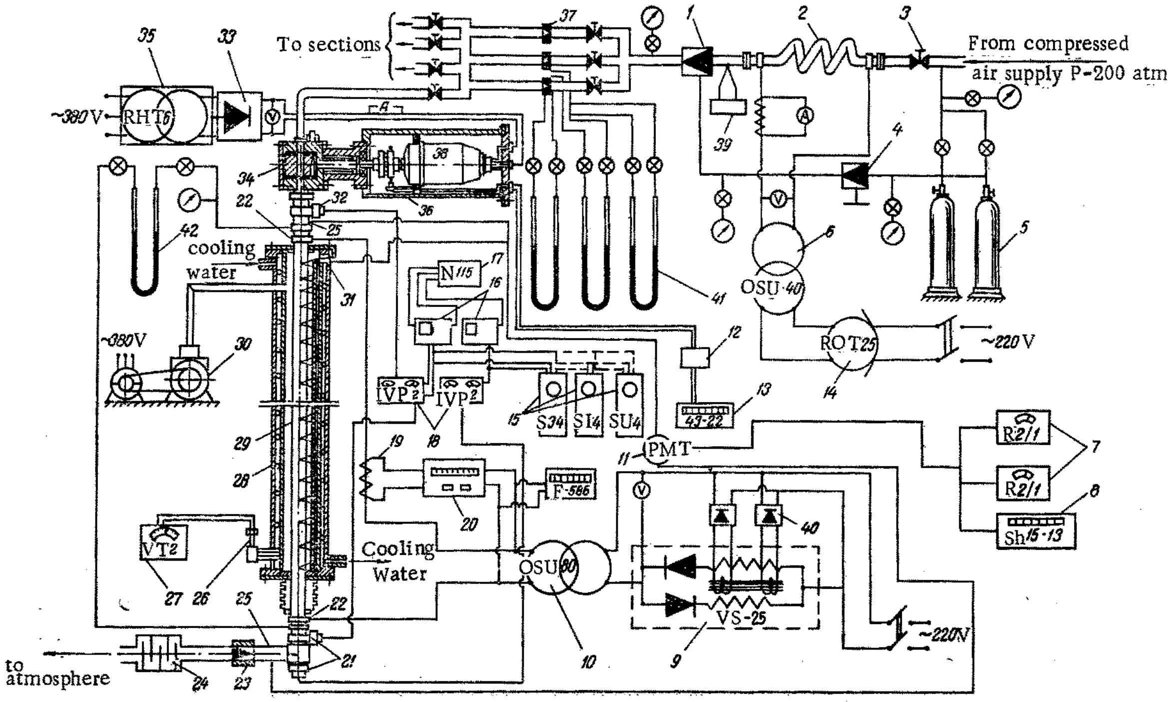

Fig. 1. Scheme of the experimental apparatus. 1, flow-rate reducer; 2, heater; 3, stopcock; 4, command reducer; 5, command air containers; 6, OSU-40 transformer; 7, R2/1 potentiometer; 8, Sh15-13 DC voltmeter; 9, USO-80 magnetic amplifier; 10, OSU-80 transformer; 11, PMT multiple switch; 12, intermediate amplifier; 13, 43-12 electronic frequency meter; 14, ROT-25 autotransformer; 15, spectrometers (SZCh, SICh, SUCh); 16, S-1-16 double-beam oscilloscopes; 17, N-115 loop oscilloscope; 18, IVP-2 amplifier-converter unit; 19, UTT-6 transformer; 20, volt-milliammeter Ø563; 21, DDI-21 pressure probes; 22, current supply; 23, throttle; 24, damper; 25, antenna thermocouple; 26, vacuum tube; 27, VT-2A vacuum meter; 28, vacuum insulation (casing); 29, test section (tube); 30, VN-2 vacuum pump; 31, plug connector; 32, DDI-20 pressure pickup; 33, VN42/70 rectifier; 34, pulsator; 35, RNT-6 autotransformer; 36, capacitance-type sensor of the number of revolutions; 37, throttle washer; 38, engine; 39, L-64 ratiometer; 40, VSA-5K rectifiers; 41,42, differential manometers.

can be written in the linear approximation as

$$\begin{gathered}\frac{\partial^2\Delta(\rho u)}{\partial\tau^2}-\frac{\partial}{\partial x}\left[(a^2-u^2)\frac{\partial\Delta(\rho u)}{\partial x}\right]+\frac{\partial}{\partial x}\left[2u\frac{\partial\Delta(\rho u)}{\partial\tau}\right]+m\frac{\partial\Delta(\rho u)}{\partial\tau}=0,\\ \frac{\partial}{\partial\tau}\left[\frac{1}{a^2}\frac{\partial\Delta P}{\partial\tau}\right]-\frac{\partial}{\partial x}\left[\left(1-\frac{u^2}{a^2}\right)\frac{\partial\Delta P}{\partial x}\right]+\frac{\partial}{\partial x}\left[\frac{2u}{a^2}\frac{\partial\Delta P}{\partial\tau}\right]+\frac{m}{a^2}\frac{\partial\Delta P}{\partial\tau}=0,\\ \frac{1}{a^2}\frac{\partial\Delta P}{\partial\tau}+\frac{\partial\Delta(\rho u)}{\partial x}=0.\end{gathered}\tag{2}$$

For sinusoidal oscillations the solution of Eqs. (2) will be sought for in the form

$$\Delta P=\Phi(x)\exp(i\omega\tau),\ \ \Delta(\rho u)=F(x)\exp(i\omega\tau).\tag{3}$$

Substituting the quantity (3) into Eqs. (2) and introducing the new variables $\xi=\int_0^x(dx)/(a^2-u^2)$ and $F=Z\exp\left[\int_0^\xi i\omega u\,d\xi\right]$ we obtain a wave equation in the variable Z,

$$\frac{d^2Z}{d\xi^2}+\omega^2a^2q(x)Z=0,\tag{4}$$

where

$$q(x) = 1 - i(1 - M^2)\left(\frac{m}{\omega} + \frac{1}{\omega}\frac{\partial u}{\partial x}\right).$$

To solve the initial equation (4) in the region of high-frequency oscillations we will use the WKB method [3]. Then, for comparatively small values of the attenuation factors, the expression for $\Phi(x)$ and $F(x)$ has the form

$$F(x) = C_{1,2}\frac{1}{\sqrt{a}}\exp\left\{\pm i\omega\int_0^x \frac{dx}{(1 \mp M)a} \mp \omega\int_0^x \alpha\,dx\right\},$$

$$\Phi(x) = \pm C_{1,2}\sqrt{a}\,\frac{1}{(1 \mp M)}\exp\left\{\pm i\omega\int_0^x \frac{dx}{(1 \pm M)a} \mp \omega\int_0^x \alpha dx\right\}, \tag{5}$$

where α is the attenuation factor of the amplitudes of the pressure and velocity oscillations. The value of the constants C_1 and C_2 is determined from the boundary conditions.

For a channel at the output of which there is a nozzle, the boundary conditions can be written in the quasilinear approximation as

$$\text{for } x = 0 \quad \Phi_{(0)} = \Delta P_0, \quad \text{for } x = L \quad \Delta P(L) = A\Delta(\rho u),$$

where $A = (Sa^2)/(S_{cr}a_{cr})$.

For these boundary conditions the expressions for the distribution of the amplitudes of the pressure and velocity oscillations along the length of the channel have the form

$$\Phi(x) = \Delta P_2\sqrt{\frac{a_i}{a_2}}\frac{(1 - M_2^2)}{(1 - M_i^2)}\left\{(1 - M_i)^2\exp[-2\bar{\alpha}(x_i - x_2)] + \right.$$

$$\left. + G^2(1 + M_i)^2\exp[2\bar{\alpha}(x_i - x_2)] + 2G(1 - M_i^2)\cos 2\omega\int_{x_i}^{x_2}\frac{1}{(1 - M_i^2)}\frac{dx}{d}\right\}^{1/2} \Big/ [(1 - M_2) + G(1 - M_2)], \tag{6}$$

$$F(x) = \frac{\Delta P_2(1 - M_2^2)}{\sqrt{a_2}\sqrt{a_i}}\cdot\frac{\left\{\exp[-2\bar{\alpha}(x_i - x_2)] + G^2\exp[2\bar{\alpha}(x_i - x_2) - 2G\cos 2\omega\int_{x_i}^{x_2}\frac{1}{(1 - M_i^2)}\frac{dx}{d}\right\}^{1/2}}{(1 - M_2) + G(1 + M_2)}, \tag{7}$$

where

$$G = \left[\frac{A}{a_2} - \frac{1}{(1 + M_2)}\right] \Big/ \left[\frac{A}{a_2} + \frac{1}{(1 - M_2)}\right]; \quad \bar{\alpha} = \left(\omega\int_x^L \alpha dx\right)\frac{1}{L - x}. \tag{8}$$

Solving Eq. (6) for $\bar{\alpha}$, we obtain an expression for the experimental determination of the mean-integral attenuation factor,

$$(L - x)\bar{\alpha} = \omega\int_x^L \alpha dx = \frac{1}{2}\ln\left[\frac{E(x)}{2(1 - M_1)^2} + \sqrt{\frac{E^2(x)}{4(1 - M_1^2)} - G^2\frac{(1 + M_1)^2}{(1 - M_1)^2}}\right], \tag{9}$$

where

$$E(x) = \left|\frac{\Delta P_4}{\Delta P_2}\right|^2\frac{a_2}{a_1^2}\frac{(1 - M_1^2)^2}{(1 - M_2^2)^2}[G(1 + M_2) + (1 - M_2)]^2 - 2G(1 - M_1^2)\cos 2\omega\left(\int_{x_2}^{x_1}\frac{dx}{(1 - M^2)a}\right). \tag{10}$$

The hydrodynamics and heat exchange were investigated under pulsating flow conditions of the heat carrier on an apparatus whose scheme is shown in Fig. 1. Air was used as the working material.

The apparatus consists of the experimental section 29, and air, water, electrical, and measuring systems. The air from the high-pressure compressed-air supply (P = 200 bars), is fed through the valve 3, the heater 2, the flow-rate reducer 1, and the throttle collar 37 to the pulsator 34, which is placed at the input to the experimental section. At the output of the experimental section there is either a nozzle or a throttle 23. By changing the diameter of the nozzle we could vary the parameters of the flow in the working section without changing its input parameters. The experimental section is provided with vacuum

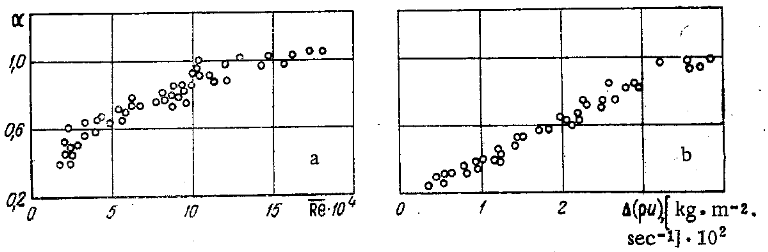

Fig. 2. Attenuation factor α (m^{-1}) as a function of the Reynolds number close to the first resonance (a), and as a function of the amplitude of the oscillation of the mass velocity $\lambda(\rho u)$ close to the first resonance (b).

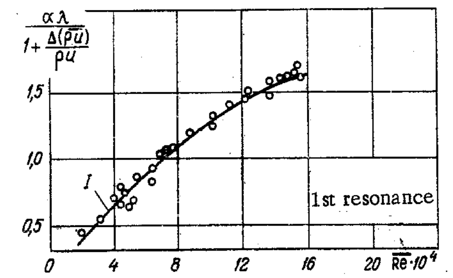

Fig. 3. Dimensionless attenuation factor times the wave length, referred to the relative amplitude of the mass velocity as a function of the Reynolds number $\overline{Re}$. I — $\alpha\lambda = [1 + \Delta(\rho u)/\rho u]\, 14.5 \cdot 10^{-4} \overline{R}^{0.586}$.

heat insulation 28. The system is evacuated by means of the VN-2A pump 30. Pulsation of the pressure and velocity of the heat carrier is achieved by means of a mechanical pulsator 34, consisting of a rotor with radial openings. The pulsator is rotated by an ST-3PT direct-current motor. The rotations and hence the pulsation frequency are controlled by means of an RNT-6 autotransformer 35. The experimental section is heated by passing a low-voltage alternating current directly through it. The power generated is controlled with a USO-80 magnetic amplifier 9. The magnetic amplifier is controlled by VSA-5K rectifiers connected in the control and bias windings. We were able to make measurements of the following quantities: the time-average air flow rate, the pressure at the input and output of the experimental section, the amplitudes of the pressure oscillations at the input and output of the experimental section, the shape of the oscillations, the pulsation frequency, the external temperature of the walls along the length of the experimental section, the electrical power generated, the heat loss and insulation, the temperature of the heat carrier at the input and output of the section, and the static pressure drop along the length of the experimental section.

To measure the amplitude and shape of the pressure oscillations we used DDI-20 and DDI-21 inductive pressure probes in the apparatus with an IVP-2 high-frequency converter.

The amplitudes of the pressure oscillations and the shape of the wave were recorded using N-115, S1-17, and S1-18 oscilloscopes, and the spectrogram of the pressure oscillations was recorded using an SZCh audiofrequency spectrometer.

The experimental apparatus consists of a calibrated tube of internal diameter 12 mm made of Kh18N9T steel. The wall thickness $\delta = 0.15$ mm and the length $L = 2045$ mm. Chambers for sampling the static pressure 12 are placed directly on the tube and are connected to a water piezometer. Pressure probes of types DDI-20 and DDI-21 are placed in the pressure sampling chambers welded to the tube. The current leads make contact with the tube through powdered graphite. The heated part of the tube of length 1286 mm is placed in vacuum insulation to reduce the loss of heat. To compensate for the vacuum and thermal displacement the body of the vacuum insulation is connected to the tube through a syphon 7. Chromel–Alumel thermocouples are placed at the entrance and exit of the experimental part. The temperature of the external surface of the heated part of the tube was measured using 24 Chromel–Alumel thermocouples.

The experimental data on the attenuation factor of the amplitudes of the pressure and velocity oscillations obtained at frequencies close to the first resonance ($f \simeq 80 \pm 5$ Hz) are shown in Figs. 2 and 3.

The mean-integral value of the attenuation factor $\overline{\alpha}$ was found from Eq. (9) from the experimentally obtained amplitudes of the pressure oscillations, the frequency, the Mach numbers, and the velocity of sound.

The attenuation factor depends, in general, on many parameters: the Mach number, the velocity of sound, the friction on the walls of the channel and in the wave itself, the velocity gradient $\partial u/\partial x$, etc. For relatively small sound-velocity gradients $\partial a/\partial x$ and small Mach numbers the attenuation factor depends on the loss due to friction on the walls of the channel and loss in the wave itself. It is not possible to distinguish experimentally between the dependence of the attenuation factor on the loss due to friction in the wave and on the walls of the channel. Equation (9) enables us to determine the value of the overall attenuation factor. In the case of low oscillation frequencies it is natural to assume that the losses due to friction on the walls of the channel have the main effect on the attenuation factor.

The attenuation factor increases as the Reynolds number of the average motion $\overline{\mathrm{Re}}$ increases (Fig. 2a). An increase in the $\overline{\mathrm{Re}}$ number is equivalent in our experiments to an increase in the amplitude of the velocity oscillation. It is probable that at the same time there is an increase in the turbulent viscosity of the heat carrier in the channel, that also leads to an increase in the viscous forces, and, consequently, to an increase in the attenuation factor.

The attenuation factor also increases as the amplitude of the oscillation of the mass velocity $\Delta(\rho u)$ increases (Fig. 2b). An increase in the amplitude of the oscillation of the mass velocity leads to a change in the turbulence spectrum on the walls of the channel and to a deformation of the boundary layer, which in turn leads to an increase in the friction loss.

For large oscillation amplitudes of the mass velocity nonlinear effects start to have an influence on the attenuation factor and lead to the appearance of harmonics of higher order than the fundamental, which are attenuated more intensively.

Hence, the experimentally obtained attenuation factor $\overline{\alpha}$ depends on the amplitude of the oscillation of the mass velocity and on the Reynolds number of the average motion.

The results of experiments on the dimensionless attenuation factor $\alpha\lambda$ for resonance oscillations close to the first resonance harmonic $f_s = 80 \pm 5$ Hz can be generalized by the relation $\alpha\lambda = f[\overline{\mathrm{Re}}, \Delta(\rho u)/\rho u]$ (Fig. 3) and approximated by the expression

$$\alpha\lambda = 14.5 \cdot 10^{-4}\left[1 + \frac{\Delta(\rho u)}{\rho u}\right]\mathrm{Re}^{0.586}.$$

NOTATION

Φ, friction force; Φ_w, friction force on the walls; Φ_f, friction force in the flow; μ, viscosity; Z, impedance; ω, angular frequency; a, velocity of sound; τ, time; u, flow velocity; M, Mach number; α, attenuation factor; P, pressure; m, loss factor; $\Phi(x)$ and $F(x)$, complex amplitudes of the compression wave ΔP and mass velocity $\Delta(\rho u)$; x, longitudinal coordinate; S, cross section of the test tube; S_{cr}, critical nozzle cross-sectional area; f, vibrational frequency; λ, oscillation wavelength. Subscripts: 1 and 2 refer to the entrance and exit of the channel section.

LITERATURE CITED

1. B. M. Galitseiskii, Yu. I. Danilov, et al., Izv. Akad. Nauk SSSR, Énergetika i Transport, No. 4 (1967).
2. B. M. Galitseiskii, Yu. I. Danilov, et al., Izv. Akad. Nauk SSSR, Énergetika i Transport, No. 5 (1968).
3. J. Heading, Introduction to the Method of Phase Integrals (the WKB method) [Russian translation], Mir (1965).